Fermi - Dirac distribution law →

In the Fermi-Dirac statistics the particles are indistinguishable but only one particle will be accupied by a single cell.
let

Energy level = $\epsilon_1, \epsilon_2 \ldots \epsilon_i \ldots \epsilon_k$
Degeneracies = $g_1, g_2 \ldots g_i \ldots g_k$
accupation no = $n_1, n_2 \ldots n_i \ldots n_k$

In the case of Fermi-Dirac statistics n_i indistinguishable particle to g_i distinguishable levels under the restrictiction that only one particle will be occupied by a single level. $g_i \geq n_i$ because there must be atleast one elementary wave function ~~avails~~ available for every element in the group.

Now the distribution of n_i particles among the g_i states can be done in the following way.

We easily find that the first particle can be put in anyone of the ith level in g_i ways. Now according to pauli exclusion principle no more particles can be assigned to that filled state. Thus we are left with $(g_i - 1)$ states in $(g_i - 1)$ ways and so on. Thus the number of ways in which n_i particles can be assigned to g_i states is

$$g_i (g_i - 1)(g_i - 2) \cdots (g_i - n_i + 1)$$

$$= \frac{g_i!}{(g_i - n_i)!}$$

The permutations among identical particles do not give distinct distributions and hence such permutations must be excluded from eqn(1) which can be done on dividing it by $n_i!$. Thus we have the required number as

$$\frac{g_i!}{n_i!(g_i - n_i)!}$$

The total number of eigen states for the whole system is given by

$$G = \prod_i \frac{g_i!}{n_i!(g_i - n_i)!}$$

The probability ω of the specific state being proportional to G will be

$$\omega = \prod_i \frac{g_i}{n_i!(g_i - n_i)!} \times \text{constant}$$

Taking log on both side

$$\log \omega = \Sigma_i \{ \log g_i! - \log n_i! - \log (g_i - n_i)! \} + \text{constant}$$

Using Stirling approximation

$$\log \omega = \sum_i \{(g_i - n_i)\log(g_i - n_i) + g_i \log g_i - n_i \log n_i\} + \text{constant}$$

differentiating it w.r.to n_i

$$\delta \log \omega = \sum_i \delta\{(g_i - n_i)\log(g_i - n_i) + g_i \log g_i - n_i \log n_i\}$$

$$= \sum \delta n_i \log(g_i - n_i) + \frac{(n_i - g_i)}{(g_i - n_i)}(-\delta n_i) - \delta n_i \log n_i - \frac{n_i}{n_i}\delta n_i\}$$

$$= \sum_i -\{\log n_i - \log(g_i - n_i)\}\delta n_i$$

$$= -\sum_i \left[\log \frac{n_i}{(g_i - n_i)}\right]\delta n_i$$

The condition of maximum probability

$$\sum_i \left[\log \left(\frac{n_i}{(g_i - n_i)}\right)\right]\delta n_i = 0$$

$\delta n = \sum \delta n_i = 0$ — (A) $\delta E = \sum \epsilon_i \delta n_i = 0$ (B)

Applying the lagrange method of ~~understand~~ undetermined multipliers ie multiplying equation by α and β

and adding the resulting expression

$$\sum_i \left[\log \frac{n_i}{(g_i - n_i)} + \alpha + \beta \epsilon_i\right] \delta n_i = 0$$

Since δn_i's can be treated as

$$\log \frac{n_i}{(g_i - n_i)} = -(\alpha + \beta \epsilon_i)$$

$$\frac{g_i}{n_i} - 1 = e^{(\alpha + \beta \epsilon_i)}$$

$$\frac{g_i}{n_i} = 1 + e^{(\alpha + \beta \epsilon_i)}$$

$$\boxed{n_i = \frac{g_i}{\left(1 + e^{\alpha + \beta \epsilon_i}\right)}}$$

This is the most probable distribution according to Fermi Dirac statistics.